Topology is the study of shapes and spaces. Specifically, it's the study of properties that don't change when shapes are stretched, such as the number of holes and twists in a figure.

1 The Mobius Strip

The discovery of the Mobius strip, or Mobius band, is often credited to the German mathematician and astronomer August Ferdinand Mobius, but the intriguing loop was actually described four years earlier by Johann Benedict Listing. The strip is an interesting mathematical figure because it has only one surface, so an ant could crawl along the length of the loop and return to its original starting point, having traversed both "sides" of the strip without ever crossing an edge.

First, take the 2 ends of the strip, twist it once, and tape the 2 sides together.

1. How many sides do you think the Mobius Strip has?
2. How many does it really have?

Answer these questions for the following parts:

1. What do you predict you will get after you cut it in half?
2. What actually happened? Why?

Now, we will cut the mobius strip in half.

Now, instead of cutting the mobius strip in half, we will cut it into thirds.

Now, instead of making a mobius strip with 1 twist before taping it, twist it twice and then tape it. We will cut it in half.

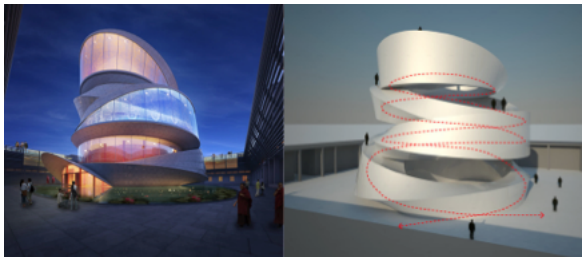

Figure 1: A Mobius Strip Temple

2 The Mobius Cross

Answer these questions for the following parts:

1. What do you predict you will get after you cut it in half?
2. What actually happened? Why?

Let's make a mobius cross, as demonstrated on the board, with no twists.

Now make a mobius cross, with one ring normal and the other ring twisted.

Now make a mobius cross, with both rings twisted.

3 Klein Bottles

Klein Bottles are a discovery of German mathematician Felix Klein and share similar characteristics to the Mobius strip. He had this idea that if you take a cylinder, turn it around, bring it through itself, and have the end welded to the base, then you can create something with one-side and no edges. They can also be formed when you connect two mobius strips side by side. However, these bottles exist only in four dimensional space or 4D. Using the ant example again, if an ant were to crawl onto a Klein Bottle, it would be able to travel in and out of the bottle without ever crossing an edge.

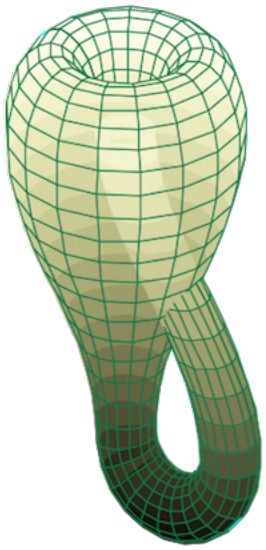

Figure 2: The Klein Bottle